Section 1.2 Addition with Whole Numbers and Perimeter

1. Review Addition Facts and Algorithm: Please review your addition facts and the addition algorithm (the process for adding numbers).

2. Vocabulary: If a and b are any two numbers, then the sum of a and b is a + b. To find the sum of two numbers, we add them.

English words	Math symbols
the sum of a and b	a+b
the sum of 4 and 8	4 + 8
the sum of y and 2	y + 2
8 more than 9	9 + 8
5 more than x	x + 5
y increased by 5	y + 5

Example: Translate each English phrase into math symbols.

a. the sum of b and 10

b + 10

b. the sum of p and q

p + q

c. p increased by 5

p + 5

d. 7 more than z

z + 7

Example: Find the sum of 10 and 18.

10 + 18 = 28

Note: Portions of this document are excerpted from the textbook *Prealgebra,* 7th ed. by Charles McKeague

3. Properties of Addition: The three properties of addition are:

- the addition property of zero
- the commutative property of addition
- the associative property of addition.

These properties are stated below. On the first test, you may be asked to state these properties in proper mathematical vocabulary. You should memorize them as given below.

Addition property of zero: If a is any number, then it is true that

$$a + 0 = 0 + a = a$$

Commutative property of addition: If a and b are any two numbers, then it is true that:

$$a + b = b + a$$

Associative property of addition: If a, b, and c are any three numbers, then it is true that:

$$(a + b) + c = a + (b + c)$$

Example: Rewrite each of the following using the addition property of zero.

a. $5 + 0 = 0 + 5 = 5$

b. $z + 0 = 0 + z = z$

Example: Rewrite each of the following using the commutative property of addition.

a. $5 + 7 = 7 + 5$

b. $z + 8 = 8 + z$

c. $2 + (4 + 5) = (4 + 5) + 2$

d. $x + (z + 9) = (z + 9) + x$

Example: Rewrite each of the following using the associative property of addition.

a. $3 + (5 + 8) = (3 + 5) + 8$

b. $z + (x + 6) = (z + x) + 6$

c. $y + (p + 4) = (y + p) + 4$

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7th ed. by Charles McKeague

Example: Name the addition property or properties used in each of the following.

a. $4 + (5 + 6) = (4 + 6) + 5$ (both comm. and assoc. prop. of addition are used since both order and grouping change)

b. $(x + 2) + 8 = x + (2 + 8)$ Associative Property of Addition

c. $y + (5 + 4) = y + (4 + 5)$ Commutative Property of Addition

d. $x + (4 + 7) = (x + 7) + 4$ Both Commutative and Associatives Properties of Addition were used

4. Solving Equations By Simplifying and Then Guessing the Answer: In the given equations, first use the properties of addition to simplify, then "guess" the solution.

Example: Simplify and solve:

a. $n + 3 = 8$

$n = 5$	guess the solution

b.

$(x + 4) + 5 = 12$	given equation
$x + (4 + 5) = 12$	associative prop. of addition
$x + 9 = 12$	addition facts
$x = 3$	guess the solution

c. $5 + (6 + x) = 14 + 2$

$(5+6) + x = 16$	Associative Property of Addition
$11 + x = 16$	Addition Facts
$x = 5$, since	$11 + 5 = 16$
	$16 = 16$ is true.

5. Polygons: A polygon is any closed geometric figure, with at least three sides, in which each side is a straight line. Figures such as rectangles, squares, triangles and octagons are all polygons. A circle is not a polygon since it doesn't have "sides" that are straight lines.

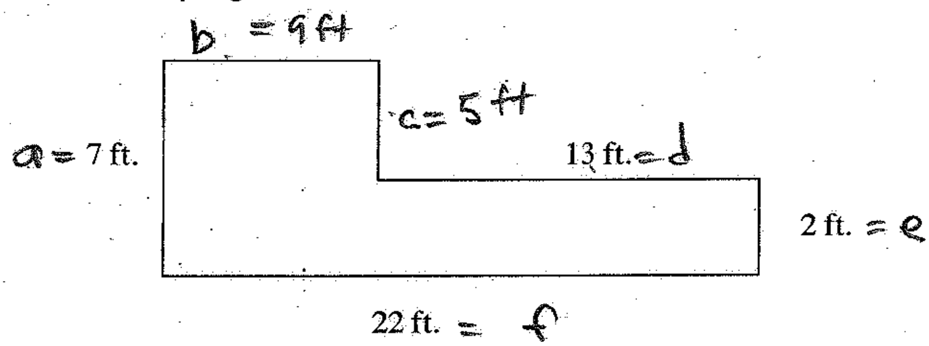
6. Perimeter: The perimeter of any polygon is the sum of the lengths of the sides, and it is denoted with the letter P. To find the perimeter of a polygon, first label the sides with letters starting with the letter a, and then write the formula for the perimeter by writing P = a + b + .. , expressing P as the sum of all of the letters representing the sides. Next plug in the known values for the sides, and lastly find the sum to get the perimeter. Be sure to attach the correct units to your answer.

Note: Portions of this document are excerpted from the textbook *Prealgebra,* 7th ed. by Charles McKeague

Example: Find the perimeter of a triangle with sides 5 ft, 11 ft. and 21 ft. Write the formula, plug in the known values, and find the perimeter.

$P = a+b+c$	Since there are three sides, you need three letters to represent the sides.
$P = 5 + 11 + 21$	Plug in the known values.
$P = 37$ ft.	Find the sum to get the perimeter. Attach the correct units.

Example: Find the perimeter of the given figure. Write the formula, plug in the known values, and find the perimeter



This figure has __6__ sides. Write the formula for the perimeter. P = a + b + c + d + e + f

Notice that the lengths of two of the sides are not given. Locate the horizontal sides. There are __3__ horizontal sides. The sum of the two short horizontal sides must equal the length of the longest horizontal side. The longest horizontal side has length __22 ft__. One of the shorter horizontal sides has length __13 ft__. What is the length of the third horizontal side?
__9 ft__

22 = b + 13
9 = b

Use this same principle to find the vertical side that does not have a given length.
Now the lengths of all the sides are known. Plug those lengths into the formula, and then add them to find the perimeter.

P = (7 ft) + (9 ft) + (5 ft) + (13 ft) + (2 ft) + (22 ft)
P = (16 ft) + (18 ft) + (24 ft)

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P = 58 ft

Section 1.3 Rounding Numbers, Estimating Answers and Displaying Information

1. Rounding numbers: In order to round a number to a given decimal place,

1. Note the decimal place in which the rounding is to occur.
2. Locate the digit just to the right of that place.
3. If that digit is less than 5, replace it and all digits to the right of it with zeros.
4. If that digit is 5 or greater, replace it and all digits to the right of it with zeros and then add 1 to the digit in the "rounding place".

Example: Round to the indicated place.

a. 2345 to the nearest ten.

$2,345 \approx 2,350$

b. 2345 to the nearest hundred.

$2,345 \approx 2,300$

c. 23,954 to the nearest hundred.

$23,954 \approx 24,000$

2. Rules for rounding in calculations: When you are completing a calculation and are asked to round your answer, do all calculations first without rounding and then round your final answer.

Example: Multiply and round to the nearest hundred.

235(446) $= 104,810$

$\approx 104,800$

$$\begin{array}{r} 235 \\ \times\, 446 \\ \hline 1410 \\ 9400 \\ 94000 \\ \hline 104,810 \end{array}$$

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7th ed. by Charles McKeague

3. Estimating: When estimating the answer to a calculation, use rounded numbers to do the arithmetic. You don't need to round your answer.

Example: Estimate the sum of the three numbers 435, 761 and 997 by rounding each number to the nearest ten.

$435 + 761 + 997 \approx 440 + 760 + 1{,}000$

$\approx 2{,}200$

$$\begin{array}{r} 440 \\ 760 \\ +1{,}000 \\ \hline 2{,}200 \end{array}$$

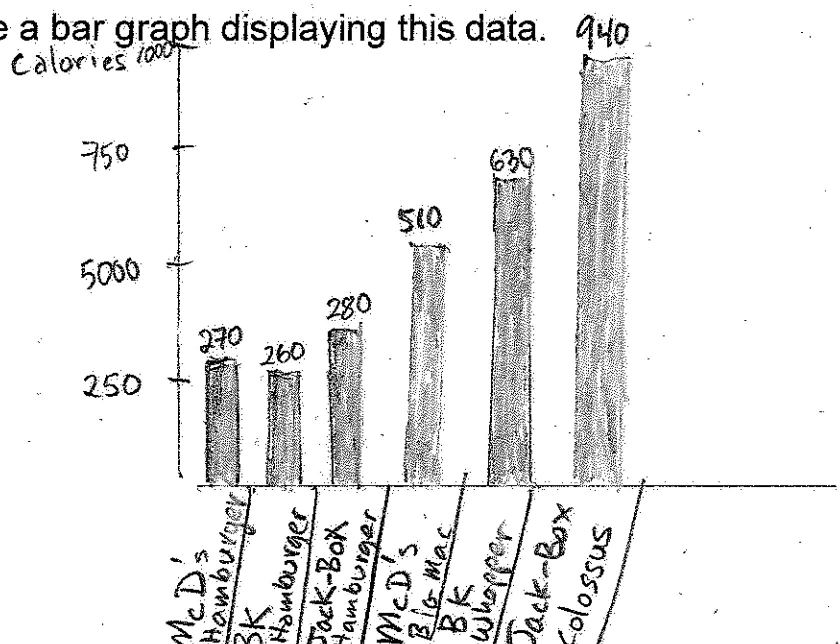
4. Displaying data: Data can by displayed using tables, pie charts, bar charts, or graphs.

Example: The following table lists the number of calories consumed by eating some popular fast foods.

Calories in Fast Food

Food	Calories
McDonald's Hamburger	270
Burger King Hamburger	260
Jack-in-the-Box Hamburger	280
McDonald's Big Mac	510
Burger King Whopper	630
Jack-in-the-Box Colossus	940

Make a bar graph displaying this data.



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Section 1.4 Subtraction with Whole Numbers

1. Vocabulary: The difference of two numbers a and b is $a-b$.

English Words	Math Symbols
The difference of 9 and 1	$9-1$
The difference of m and 4	$m-4$
The difference of 4 and m	$4-m$
3 subtracted from m	$m-3$
5 decreased by x	$5-x$
x subtracted from y	$y-x$

2. Subtraction algorithm: Please review subtraction facts and the subtraction algorithm, including the borrowing technique.

Example: Simplify.

a. $2462-584 = 1{,}878$

spwk

$$\begin{array}{r} {\scriptstyle 1\ 13\ 15\ 12} \\ \cancel{2}\cancel{4}\cancel{6}\cancel{2} \\ -\ 584 \\ \hline 1{,}878 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\ \ 1\ 1} \\ 1878 \\ +\ 584 \\ \hline 2{,}462 \end{array}$$

chek ✓

b. $457-298 = 159$

spwk

$$\begin{array}{r} {\scriptstyle 3\ 14\ 17} \\ \cancel{457} \\ -\ 298 \\ \hline 159 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\ 1} \\ 298 \\ +159 \\ \hline 457 \end{array}$$

chek ✓

Section 1.5 Multiplication with Whole Numbers

1. Definition of Multiplication and Multiplication Notation: Multiplication is repeated addition. Thus, 5 times 8 means 8 + 8 + 8 + 8 + 8. To denote multiplication, you may use a dot, parenthesis or 'x'. Since we often use 'x' for a variable in algebra, the dot or parenthesis notation is preferable.

To write 3 times 4 in math notation, you can write

$3 \bullet 4$, $3(4)$, $(3)(4)$, or 3×4

2. Vocabulary: The product of two numbers a and b is $a \bullet b$. The "a" and "b" are called factors. The product may be written "ab" (omit the dot between the letters) if the at least one of the factors is a variable.

English Words	Math Symbols
The product of 2 and 5	$2 \cdot 5$
The product of 2 and y	$2y$
The product m and n	mn
The product of x and 2	$2x$
The product of p and q	pq

3. The Distributive Property: If a, b and c are any numbers, then

$$a(b+c) = ab + ac$$

Example: Use the distributive property to simplify:

a. $5(4+3) = 5 \cdot 4 + 5 \cdot 3 = 20 + 15 = 35$

b. $7(2+6) = 7 \cdot 2 + 7 \cdot 6 = 14 + 42 = 56$

c. $2(x+3) = 2 \cdot x + 2 \cdot 3 = 2x + 6$

d. $3(y+7) = 3 \cdot y + 3 \cdot 7 = 3y + 21$

e. $5(m+3) = 5 \cdot m + 5 \cdot 3 = 5m + 15$

f. $6(x+6) = 6 \cdot x + 6 \cdot 6 = 6x + 36$

g. $8(21+10) = 8 \cdot 21 + 8 \cdot 10 = 168 + 80 = 248$

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When expressions such as those above contain “like quantities” inside the parenthesis, there are two ways to proceed in simplifying:

- you can first perform the addition or subtraction inside the parentheses, and then perform the multiplication, or
- you can use the distributive property, and then simplify.

However, if the quantity inside the parenthesis contains “unlike quantities”, you can’t do the addition or subtraction first, so you must use the distributive property in order to simplify.

Example: Simplify each of the following by adding or subtracting first, then performing the multiplication.

a. $5(4+3) = 5 \cdot 7 = 35$

b. $7(2+6) = 7 \cdot 8 = 56$

c. $8(21+10) = 8 \cdot 31 = 248$

4. Geometric Representation of the Distributive Property: The distributive property can be shown geometrically as follows:

Example: Show the distributive property geometrically.

a. $3(4+5) = 3 \bullet 4 + 3 \bullet 5$

Use 3 rows, 4 X’s and 5 O’s

$$\begin{bmatrix} X X X X O O O O O \\ X X X X O O O O O \\ X X X X O O O O O \end{bmatrix} = \begin{bmatrix} X X X X \\ X X X X \\ X X X X \end{bmatrix} + \begin{bmatrix} O O O O O \\ O O O O O \\ O O O O O \end{bmatrix}$$

b. $5(2+6)$

$$\begin{bmatrix} X X O O O O O O \\ X X O O O O O O \\ X X O O O O O O \\ X X O O O O O O \\ X X O O O O O O \end{bmatrix} = \begin{bmatrix} X X \\ X X \\ X X \\ X X \\ X X \end{bmatrix} + \begin{bmatrix} O O O O O O \\ O O O O O O \\ O O O O O O \\ O O O O O O \\ O O O O O O \end{bmatrix}$$

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5. Distributive Property and the Multiplication Algorithm: The multiplication algorithm is based on the distributive property. To see this, consider the following example:

Example: Simplify.

a. $25(12) = 25(10+2)$

$$= 25 \bullet 10 + 25 \bullet 2$$
$$= 250 + 50$$
$$= 300$$

Example: Multiply the following numbers in your head.

a. 25(14) $= 25(10+4) = 25 \cdot 10 + 25 \cdot 4 = 250 + 100 = 350$

b 15(12) $= 15(10+2) = 15 \cdot 10 + 15 \cdot 2 = 150 + 30 = 180$

c. 22(13) $= 22(10+3) = 22 \cdot 10 + 22 \cdot 3 = 220 + 66 = 286$

6. Multiplication Algorithm: Please review multiplication facts and the multiplication algorithm.

Example: Find the product of 849 and 76

$849 \cdot 76 = 64{,}524$

scratch work

36
~~25~~
849
× 76
5094
+59430
64,524

7. Properties of Multiplication: The four properties of multiplication are:

- the multiplication property of zero
- the multiplication property of one
- the commutative property of multiplication
- the associative property of multiplication.

These properties are stated below. On the first test, you may be asked to state these properties in proper mathematical vocabulary. You should memorize them as given below:

Multiplication property of zero: If a is any number, then

$$a \bullet 0 = 0 \bullet a = 0$$

Multiplication property of one: If a is any number, then

$$a \bullet 1 = 1 \bullet a = a$$

Commutative property of multiplication: If a and b are any numbers, then

$$a \bullet b = b \bullet a$$

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Associative property of multiplication: If a, b and c are any numbers, then

$$a(bc) = (ab)c$$

Example: Name the property or properties that are illustrated in each of the following.

a. $3(4 \bullet 7) = (3 \bullet 4) \bullet 7$, Associative Property of multiplication

b. $4(8 \bullet 9) = 4(9 \bullet 8)$, Commutative Property of multiplication

c. $5 \bullet 0 = 0 \bullet 5 = 0$, Multiplication Property of Zero

d. $6(7 \bullet 2) = (6 \bullet 2) \bullet 7$, Both the Commutative Property of Mult. & the Associative Property of Mult.

8. Solving Equations by Inspection:

Example: Solve by inspection.

a. $5n = 25$

$n = 5$ (guess the solution)

$5 \cdot (5) = 25$
$25 = 25$ is true!

b. $7n = 28$

$n = 4$

$7 \cdot (4) = 28$
$28 = 28$ is true!

9. Multiplying by Powers of Ten: To multiply by a power of ten, count the number of zeros on the power of ten and place that many zeros on the end of the other factor to get the answer.

Example:

a. $100 \bullet 27 = 2700$

b. $10{,}000 \bullet 82$ = 820000

c. $1{,}000{,}000 \bullet 132$ = 132,000,000

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7th ed. by Charles McKeague

To multiply by a number that ends in zeros:

1. Drop the zeros.
2. Multiply the two remaining numbers.
3. Place the zeros at the end of the product.

Example:

a. $2700 \bullet 3 = 8100$

b. $42 \bullet 2,\underline{000}$ = 84,000

c. $32,\underline{000} \bullet 8\underline{0}$ = 2,560,000

Scwk

$$\begin{array}{r} 42 \\ \times 2 \\ \hline 84 \end{array}$$

$$\begin{array}{r} \overset{1}{3}2 \\ \times 8 \\ \hline 256 \end{array}$$

Note: Portions of this document are excerpted from the textbook *Prealgebra,* 7th ed. by Charles McKeague

Section 1.6 Division with Whole Numbers

1. Notation: Division can be written in four ways. For example, to write ten divided by five, we could write any of the following:

$10 \div 5$ or $\frac{10}{5}$ or $10/5$ or $5\overline{)10}$

2. Vocabulary: The quotient of a and b is

$a \div b$ (or $\frac{a}{b}$ or a/b or $b\overline{)a}$).

The number that you are dividing into, the "a" in this case is the dividend, the number that you are dividing by, the "b" in this case is the divisor, and the quotient is $a \div b$ (or $\frac{a}{b}$ or a/b or $b\overline{)a}$).

English words	Math symbols
10 divided by 5	$10 \div 5$
23 divided by 8	$23 \div 8$
8 divided by 25	$8 \div 25$
x divided by 5	$x \div 5$
the quotient of 12 and 3	$12 \div 3$
the quotient of 12 and x	$12 \div x$

3. The Meaning of Division

Division is repeated subtraction. Twelve divided by four is three since $12 - 4 - 4 - 4 = 0$. In words, you can subtract three fours from twelve before you get to a number that is smaller than four. The last number, the one that is smaller than four is called the remainder. In this case the remainder is zero.

You can check any division problem by multiplication.

$35 \div 4 = 8$ R.3, so $8 \bullet 4 + 3 = 35$

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4. Division Algorithm: Please review the division algorithm on page 58 of your textbook.

Example: Simplify.

a. $4606 \div 49$ = 94

SDWK

$$\begin{array}{r} 94 \\ 49\overline{)4606} \\ -441 \\ \hline 196 \\ -196 \\ \hline 0 \end{array} \qquad \begin{array}{r} 49 \\ \times 9 \\ \hline 441 \end{array} \qquad \begin{array}{r} 49 \\ \times 4 \\ \hline 196 \end{array} \qquad \begin{array}{r} 49 \\ \times 94 \\ \hline 196 \\ +4410 \\ \hline 4606 \end{array}$$

b. $\dfrac{17595}{45}$ = 391

$$\begin{array}{r} 391 \\ 45\overline{)17595} \\ -135 \\ \hline 409 \\ -405 \\ \hline 45 \\ -45 \\ \hline 0 \end{array} \qquad \begin{array}{r} 45 \\ \times 3 \\ \hline 135 \end{array} \qquad \begin{array}{r} 45 \\ \times 9 \\ \hline 405 \end{array} \qquad \begin{array}{r} 391 \\ \times 45 \\ \hline 1955 \\ +15640 \\ \hline 17595 \end{array}$$

5. Division by Zero: Division by zero is undefined, but zero divided **by** any number other than zero is zero. Thus,

$0 \div 5 = 0$ and $5 \div 0$ is undefined.

Example: Simplify:

a. $8 \div 0$ is undefined

b. $16 \div 0$ is undefined

c. $0 \div 12$ = 0

d. $\dfrac{13}{0}$ is undefined

e. $\dfrac{0}{2}$ = 0

Example: Write a division expression that gives 0 as an answer.

$0 \div 4 = 0$

Example: Write a division expression that is not defined.

$4 \div 0$ is undefined

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1.7 Exponents, Order of Operations and Averages

1. Exponents: An exponent is a number that indicates how many times the base is to be used as a factor. Exponents indicate repeated multiplication. The base is the number that is multiplied.

Example: Identify the base and the exponent: 2^3

Two is the base and three is the exponent.

Example: Simplify.

a. $3^2 = 3 \cdot 3$
$= 9$

b. $2^3 = 2 \cdot 2 \cdot 2$
$= 4 \cdot 2$
$= 8$

c. $4^3 = 4 \cdot 4 \cdot 4$
$= 16 \cdot 4$
$= 64$

SDWK
$$\begin{array}{r} \overset{2}{16} \\ \times 4 \\ \hline 64 \end{array}$$

d. $5^2 = 5 \cdot 5$
$= 25$

2. Zero exponent: Any number other than 0 raised to the zero power is 1.

Example: Simplify.

a. $2^0 = 1$

b. $15^0 = 1$

c. x^0 (assume that x is not zero) $x^0 = 1$

Example: Write an expression that contains an exponent and that has a value of 1.

$25{,}027^0 = 1$

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3. Order of operations: When evaluating mathematical expressions, we will perform operations in the following order:

First: If the expression contains grouping symbols, such as parenthesis (), brackets [ ], braces { }, or a fraction bar, then we perform the operations inside the grouping symbols, or above and below the fraction bar, first.

Second: Evaluate, or simplify, any numbers with exponents.

Third: Do all multiplications and divisions in order from left to right.

Fourth: Do all additions and subtractions in order from left to right.

Example: Simplify.

a. $2+3\bullet 5$ (operations of addition and multiplication are present)
$=2 + 15$ (perform multiplication first)
$=17$ (perform addition second)

b. $3+2\bullet 4^2$ (operations of addition, mult, and exponentiation)
$=3+2\bullet 16$ (do exponents first)
$=3+32$ (do multplication second)
$=35$ (do addition third)

c. $2\bullet 3+4(8-3) = 2\cdot 3 + 4(5)$
$= 6+4\cdot 5$
$= 6+20$
$= 26$

d $\dfrac{4+6\bullet 3}{15-(9-5)}$
$= \dfrac{4+18}{15-(4)}$
$= \dfrac{22}{11}$
$= 2$

e. $3^2+2(17-3^2)$
$= 3^2 + 2(17-9)$
$= 3^2 + 2(8)$
$= 9 + 2\cdot 8$
$= 9 + 16$
$= 25$

SDWK
$3^2 = 3\cdot 3$
$= 9$

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7th ed. by Charles McKeague

4. Averages: There are three types of averages: the mean, the median and the mode.
Mean: To find the mean for a set of numbers, we add all the numbers and then divide the sum by the number of numbers in the set. The mean is sometimes called the arithmetic mean.
Median: To find the median for a set of numbers, we write the numbers in order from smallest to largest. If there is an odd number of numbers, the median is the middle number. If there is an even number of numbers, then the median is the mean of the two numbers in the middle.
Mode: The mode for a set of numbers is the number that occurs most frequently. If all the numbers in the set occur the same number of times, there is no mode.

Example: Find the mean, median and mode of the set of numbers: \$35,344; \$38,290; \$39,199; \$40,346; \$42,866.

mean $= \dfrac{35{,}344 + 38{,}290 + 39{,}199 + 40{,}346 + 42{,}866}{5}$

mean $= \dfrac{\$196{,}045}{5}$

Mean $= \$39{,}209$

No Mode!

$\$35{,}344 \leq \$38{,}290 \leq \$39{,}199 \leq \$40{,}346 \leq \$42{,}866$

median (middle)

The median is \$39,199.

Example: Find the mean, median, and mode of the set of numbers: 77, 87, 100, 65, 79, 87, 79, 85, 87, 95, 56, 87, 56, 75, 79, 93, 97, 92

Mean $= \dfrac{77+87+100+65+79+87+79+85+87+95+56+87+56+75+79+93+97+92}{18}$

mean $= \dfrac{1476}{18}$

Mean $= 82$

$56 \leq 56 < 65 < 75 < 77 < 79 \leq 79 \leq 79 < 85 < 87 \leq 87 \leq 87 \leq 87 < 92 < 93 < 95 < 97 < 10$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

Median $= \dfrac{85+87}{2}$

median $= \dfrac{172}{2}$

Median $= 86$

The mode is 87, as it occurs 4 times.

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5. Vocabulary: We will now translate into mathematical symbols English phrases that contain complicated expressions involving the terms sum, product, difference, and quotient.

English phrase	Math symbols
sum of a and b	$a+b$
two times the sum of a and b	$2\cdot(a+b)$
product of p and q	pq
product of p and the sum of a and b	$p\cdot(a+b)$
sum of p and the product of a and b	$p+a\cdot b$
difference of p and the sum of a and b	$p-(a+b)$
sum of the product of a and b and the product of c and d	$a\cdot b+c\cdot d$

Example: Translate each phrase into math symbols.

a. product of 4 and the sum of 3 and x

$4\cdot(3+x)$

b. difference of 4 and the sum of 3 and x

$4-(3+x)$

c. sum of the product of 4 and 3 and the product of 2 and 5

$4\cdot 3+2\cdot 5$

d. twice the product of 4 and x

$2\cdot(4x)$

e. twice the sum of 8 and 5

$2\cdot(8+5)$

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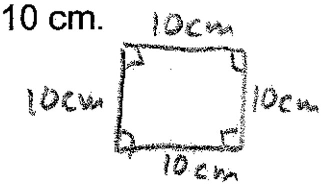
1.8 Area and Volume

1. Area: The area of a flat object is the amount of surface that the object has. Formulas for the area of some common shapes are given below. You must be able to recall and use these formulas. Units for all areas are always square units, such as in^2, ft^2, m^2, etc.

Square:	A = (side)(side) = s^2
Rectangle:	A = (length)(width) = lw
Parallelogram:	A = (base)(height) = bh

Example: Find the area of each figure. Write the formula first, plug in the known values, and then perform the computations to find the area.

a. A square with side 10 cm.

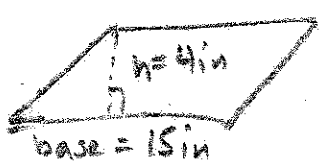


$A = s^2$

$A = (10\,cm)^2$

$A = 100\,cm^2$

b. A parallelogram with base 15 in. and height 4 in.

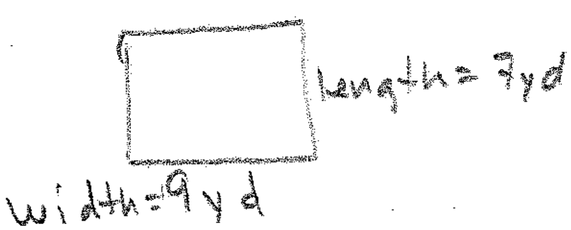


$A = bh$

$A = (15in)(4in)$

$A = 60\ in^2$

c. A rectangle with length 7 yd. and width 9 yd.



$A = lw$

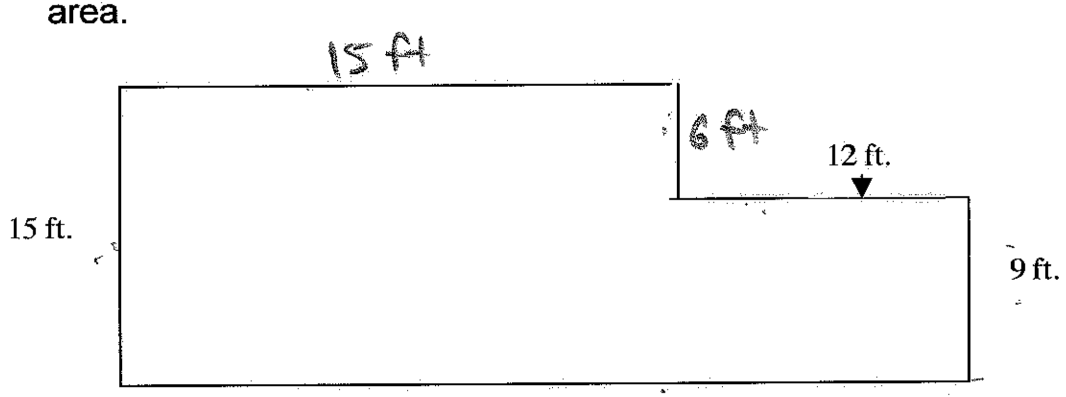
$A = (7yd)\cdot(9yd)$

$A = 63\ yd^2$

Note: Portions of this document are excerpted from the textbook *Prealgebra,* 7th ed. by Charles McKeague

2. Areas of Composite Figures: Some figures that are not squares, rectangles or parallelograms can be broken into pieces each of which is a square, rectangle or parallelogram. These figures are called composite figures. The area of the composite figure can be found be adding together the areas of the pieces.

Example: Find the area of the composite figure given below. Be sure to use proper format for showing your work: write the formula(s), plug in the known numbers and then compute the area.



27 = 12 + 15

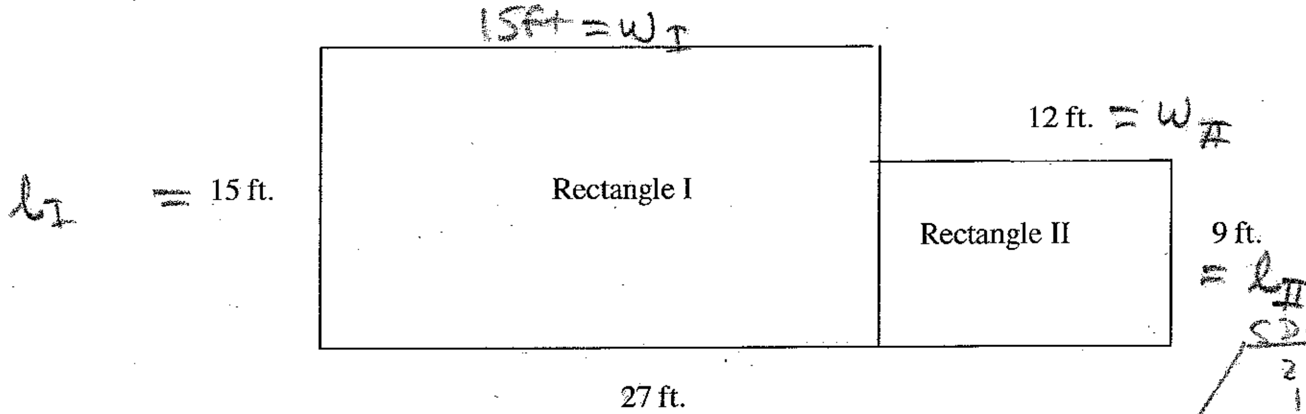
This figure has __6__ sides. Notice that the lengths of two of the sides are not given. Locate the horizontal sides. There are __3__ horizontal sides. The sum of the two short horizontal sides must equal the length of the longest horizontal side. The longest horizontal side has length __27 ft__. One of the shorter horizontal sides has length __12 ft__. What is the length of the third horizontal side? __15 ft__

Now, use this same principle to find the vertical side that does not have a given length.

15 ft = 9 ft + 6 ft

Note: Portions of this document are excerpted from the textbook *Prealgebra,* 7th ed. by Charles McKeague

Now that we know the lengths of all six sides, we must break the figure up into squares, rectangles and/or parallelograms and find the area of each piece. The sum of these areas is the area of the given figure.



SDWK

$$\begin{array}{r} 15 \\ \times 15 \\ \hline 75 \\ +150 \\ \hline 225 \end{array} \qquad \begin{array}{r} 225 \\ +108 \\ \hline 333 \end{array}$$

Area = Area Rectangle I + Area Rectangle II

$= A_I + A_{II}$

$= l_I w_I + l_{II} w_{II}$

$= (15ft)(15ft) + (9ft)(12ft)$

$= 225\ ft^2 + 108\ ft^2$

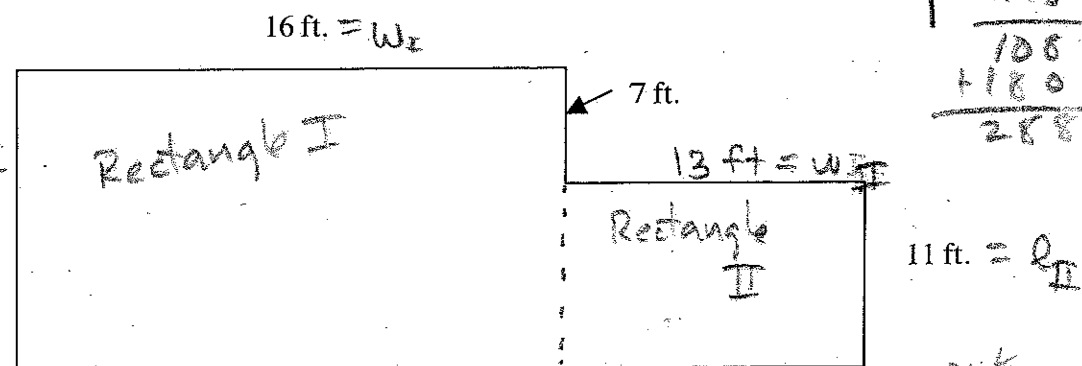
$A = 333 ft^2$

Example: Find the area of the given composite figure. Use the proper format for showing your work.

SDWK

$$\begin{array}{r} 18 \\ \times 16 \\ \hline 108 \\ +180 \\ \hline 288 \end{array}$$

$l_I = 18ft$



Total Area $= A_I + A_{II}$

$= l_I w_I + l_{II} w_{II}$

$= (18ft)(16ft) + (11ft)(13ft)$

$= 288\ ft^2 + 143\ ft^2$

$A = 431\ ft^2$

OWK

$$\begin{array}{r} 13 \\ \times 11 \\ \hline 13 \\ +130 \\ \hline 143 \end{array}$$

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7th ed. by Charles McKeague

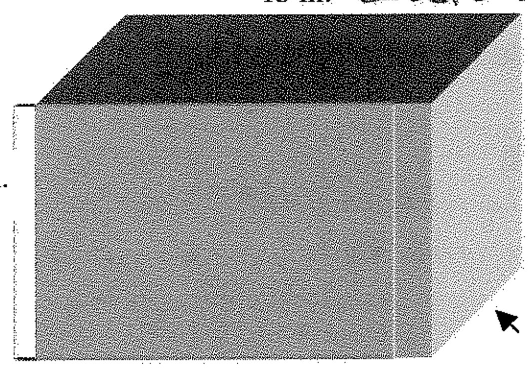
3. Volume: Volume is the measure of the space enclosed by a solid. A rectangular solid is a solid in which opposite sides are parallel and in which sides meet at right angles. The formula for the volume of a rectangular solid is:

$$V = (\text{length})(\text{width})(\text{height}) = lwh$$

The units of volume are always cubic units, such as in^3, ft^3, m^3, etc. If you look at any corner of a rectangular solid, there are three edges that come into that corner. Name any one of these edges the length, another the width and the third the height.

Example: Find the volume of the rectangular solid. Be sure to use proper format for showing your work: write the formula(s), plug in the known numbers and then compute the area.

15 m. = width



height = 17 m.

11 m. = length

$V = lwh$

$V = (11m)(15m)(17m)$

$V = 165 \cdot 17\ m^3$

$V = 2,805\ m^3$

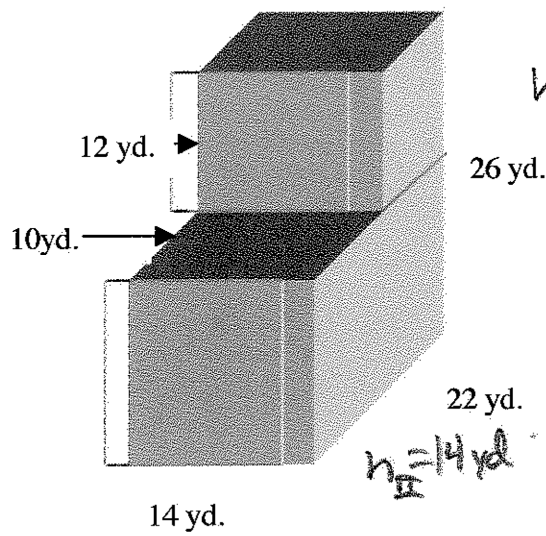
spwk

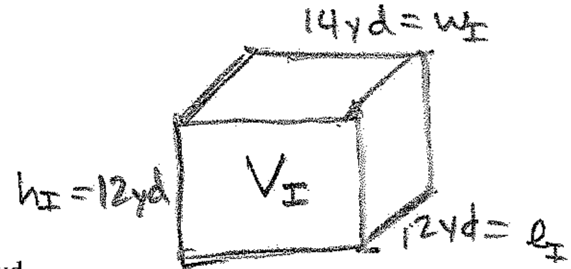
$$\begin{array}{r} 15 \\ \times 11 \\ \hline 15 \\ +150 \\ \hline 165 \end{array} \qquad \begin{array}{r} 165 \\ \times 17 \\ \hline 1155 \\ +1650 \\ \hline 2805 \end{array}$$

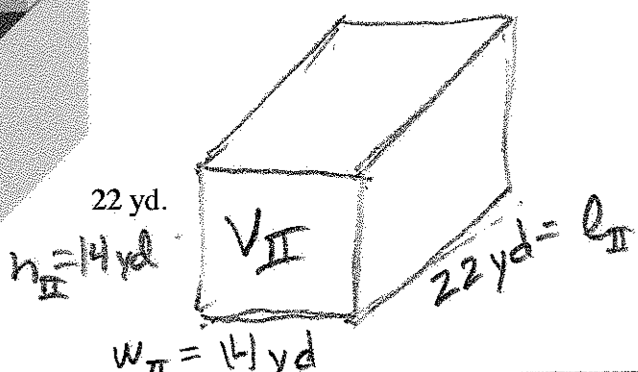
Note: Portions of this document are excerpted from the textbook *Prealgebra,* 7th ed. by Charles McKeague

Solids can be composed of several rectangular solids that are "stacked" together. To find the volume of such a composite figure, break it up into pieces that are rectangular solids, find the volume of the rectangular solids, and add those volumes together to find the volume of the composite solid.

Example: Find the volume. Be sure to use proper format for showing your work: write the formula(s), plug in the known numbers and then compute the area.







Total Volume $= V_I + V_{II}$

$= l_I w_I h_I + l_{II} w_{II} h_{II}$

$= (12yd)(14yd)(12yd) + (22yd)(14yd)(14yd)$

$= 168 \cdot 12\ yd^3 + 308 \cdot 14\ yd^3$

$= 2{,}016\ yd^3 + 4{,}312\ yd^3$

$V = 6{,}328\ yd^3$

Spwk

14×12: 28, +140, 168

168×12: 336, +1680, 2,016

22×14: 88, +220, 308

308×14: 1232, +3080, 4,312

2,016 + 4,312 = 6,328

Note: Portions of this document are excerpted from the textbook *Prealgebra,* 7th ed. by Charles McKeague